ENGINEERING MECHANICS

Notes by-

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II

s = distance
t = time
u = velocity (s/t)
a = accelⁿ
u = Initial vel.
v = Final vel.

KINEMATICS OF PARTICLES

Distinctly make Kinematics [Why?] and Kinetics [How?] clear through the concept of matter, space and time (what we engineers know as the fundamental dimensions MLT).

Why Kinematics? only for analyzing Kinetics.

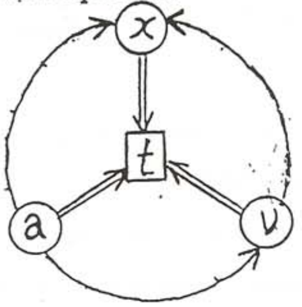
Why Kinetics is important for Engineers?

Impress on the fact that, in a theoretical sense, no particle or rigid body ever moves with constant acceleration and hence avoid the gross abuse of the three 'devilish' equations $\boxed{s = ut + \frac{1}{2} at^2}$, $\boxed{v = u + at}$ and $\boxed{v^2 = u^2 + 2as}$ in *Rectilinear motion.* Any [MLT] parameter/phenomenon in this Universe can only be a function of space or time or both. Acceleration has to be a function of position (space), velocity (space derivative), or time, or a combination of these but never a constant.

[MLT]
s - dist
$\frac{ds}{dt} = v$
$\frac{dv}{dt} = a$
$a \neq$ constant

Encourage the use of definite integrals rather than blindly using Constants while performing integration.

Convey the idea that diagrams / figures are much more powerful than equations both from the intuitive point of view as well as in simplicity. Hence encourage the students to make sketches for the solution of every problem and make them realize the superiority of motion curves over motion equations.



'radial' relations are primary motion equations [x = f (t), v = f(t), a = f(t)] more commonly known as equations of motion.
'Peripheral' relations are secondary equations of motion
[a = f(v), v = f(x), a = f(x)]

The graphical display of 'equations of motion' and 'secondary equations of motion' are respectively known as 'curves of motion' or motion curves and 'secondary curves of motion' or secondary motion curves. Secondary motion curves are more practically useful (see the second problem on page 6) than motion curves in motion analysis.

Prefer the use of *definite integrals* rather than using constants of integration. This will reduce the mathematical labour in the solutions of many problems.

In dependent motion, choose a fixed point on the line of motion. The velocity and acceleration equations, obtained on differentiating the geometry equations are, strictly not vector equations. This idea has not been conveyed through the Books of even some renowned authors but will become clear if you solve Prob. 82. of 'A collection of Problems' (ACOP).

Before starting *Curvilinear motion,* give a concrete idea about the concepts of *curvature, circle of curvature, radius of curvature,* and *centre of curvature.* Explain the significance of *point of inflection* and clearly give the mathematical explanation. After this explain the concept of rate of change of vectors in space and thereafter the relationship between a *space derivative* and *time derivative.*

Clearly understand the difference between the *magnitude of the derivative* and *derivative of the magnitude.* For example, in the third problem on page 3,

$$V = \left|\frac{d}{dt}\ \bar{r}\right| = \sqrt{(2t-8)^2 + (t+2)^2}$$

$$= \sqrt{5t^2 - 28t + 68}$$

This is the speed which is the *magnitude of the derivative.*

Now consider $\bar{r} = (t^2-8t+7)\ \bar{i} + (.5\ t^2 + 2t - 4)\ \bar{j}$

$$|\bar{r}| = \sqrt{(t^2-8t+7)^2\bar{i} + (0.5t^2+2t-4)^2\bar{j}}$$ where $|\bar{r}|$ is clearly a scalar.

$\frac{d}{dt}|\bar{r}|$ is the *derivative of the magnitude* and is the rate at which the length of the position vector changes.

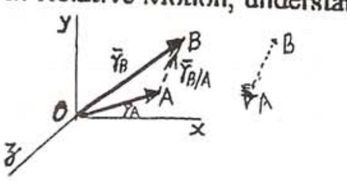
Appraise the students that the direction perpendicular to the osculating plane (the plane containing e_t and e_n directions) is known as the binormal direction even though it may be sometime referred as the z direction.

As an instructor, have a through understanding of *hodographs* as this helps in understanding how the velocity and acceleration vectors are related.

Derive the vector equations for position, velocity, and acceleration vectors in rectangular coordinates (x-y), path coordinates (n-t), and polar coordinates (r - θ) successively and thereafter the conversions of these quantities from one system to another.

Never assign *Projectile motion* any special status. This has become a malignant cancer for the last few decades among the mediocre authors who assign a so called Chapter on this topic in their books on 'Engineering Mechanics'. Projectile motion is only a special case of *curvilinear motion* that is dealt with in x-y coordinates. Give stress on the importance of the equation of trajectory as this is the most powerful equation in Projectile motion. This is simply because of fact that it is derived from two space-time relations, viz, x = f(t) and y = f(t).

In Relative Motion, understand $r_{B/A}$ as what an observer sees B from A.



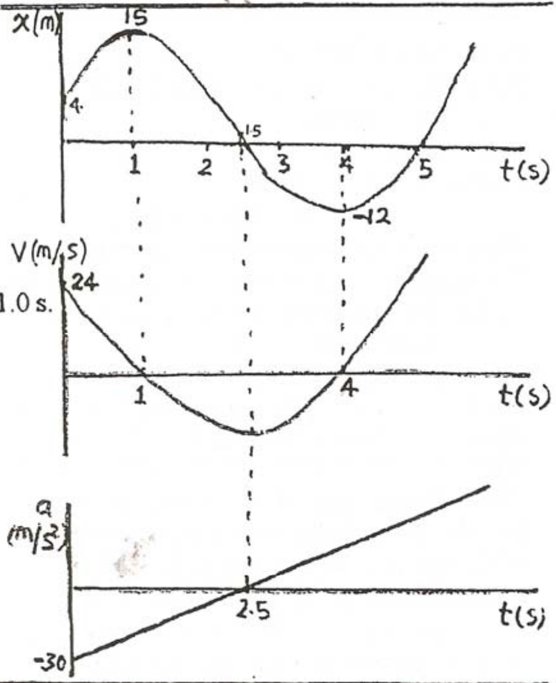
$\bar{r}_{B/A} = \bar{r}_B - \bar{r}_A \Rightarrow \bar{V}_{B/A} = \bar{V}_B - \bar{V}_A$ and $\bar{a}_{B/A} = \bar{a}_B - \bar{a}_A$.

[O is the origin of a fixed frame of reference]

Kolhe Pravin

KINEMATICS OF PARTICLES

Prob.: The motion of a particle to defined by the relation $x=2t^3 - 15t^2 + 24t + 4$, where x is expressed in meters and t in seconds. Determine (a) when the velocity is zero, (b) the position and the total distance traveled when the acceleration is zero.



Given $x = 2t^3 - 15t^2 + 24t + 4$

$\therefore v = dx/dt = 6t^2 - 30t + 24$ and $a = dv/dt = 12t - 30$

Velocity (v) is zero when $6t^2 - 30t + 24 = 0$ or $6(t-1)(t-4) = 0$

Thus velocity is zero when **t = 1s and t = 4s.**

Acceleration (a) is zero when $12t - 30 = 0$ or **t = 2.5s**

$x_{t=2.5} = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4 = 1.5$ m.

Thus the position when acceleration is zero is **1.5 m.**

To find the total distance traveled, we see whether the direction of motion changes before t = 2.5 s. It does at t = 1.0 s.

$\therefore$ Total distance covered is the sum of $\int_0^1 v dt$ and $\int_1^{2.5} v dt$

dropping their signs.

Thus $\mathbf{S} = \left|\int_0^1 v dt\right| + \left|\int_1^{2.5} v dt\right|$

$= |x_1 - x_0| + |x_{2.5} - x_1|$

$= 11 + 13.5 \qquad \mathbf{S = 24.5\ m.}$

The motion curves given, explain the details.

Prob.: The acceleration of a particle falling through the atmosphere is defined by the relation $a = g(1 - k^2v^2)$. If the particle starts at t = 0 with no initial velocity, (a) show that the velocity at time t is $v = (1/k)\tanh kgt$, (b) write an equation defining the velocity of the particle for any value of the distance x through which it has fallen. (c) Why is $v_t = (1/k)$ called the terminal velocity?

Given that $a = dv/dt = g(1 - k^2v^2) \Rightarrow \frac{dv}{(1-k^2v^2)} = g \cdot dt \Rightarrow \int \frac{dv}{k^2\left[\left(\frac{1}{k}\right)^2 - v^2\right]} = \int g\, k^2 dt$

$$\therefore \int \frac{dv}{k^2\left[(1/k)^2 - v^2\right]} = \int g dt$$

$$\boxed{\int \frac{dx}{(a^2 - x^2)} = \frac{1}{2a} \ln\left[\frac{a+x}{a-x}\right]}$$

† On integration $\frac{1}{2(1/k)} \ln\left(\frac{(1/k)+v}{(1/k)-v}\right) = gk^2t + C$

$\therefore \ln\left[\frac{1/k + v}{1/k - v}\right] = \frac{2}{k} \cdot g k^2 t \Rightarrow \ln\left[\frac{1+vk}{1-vk}\right] = 2gkt$

But at t = 0, v = 0. This implies that C = 0. This simplifies the equation to $\frac{1+kv}{1-kv} = e^{2gkt}$ and on further

simplification $v = \frac{1}{k}\left(\frac{e^{2gkt} - 1}{e^{2gkt} + 1}\right)$ or $v = \frac{1}{k}\left(\frac{e^{gkt} - e^{-gkt}}{e^{gkt} + e^{-gkt}}\right)$

$\frac{1+ax}{1-ax} = b \Rightarrow x = \frac{1}{a}\left(\frac{b-a}{b+a}\right)$

$1 + ax = b - abx \Rightarrow 1 + x(a+ab) = b$

$\Rightarrow x = \frac{b-1}{a(b+1)}$

†† or $\mathbf{v = \frac{1}{k}\tanh(gkt)}$ $\qquad \tanh\theta = \frac{e^{\theta} - e^{-\theta}}{e^{\theta} + e^{-\theta}}$

As t takes a large value, mathematically as $t \to \infty$, $\tanh(gkt)$ tends to a finite limit i.e. unity. Thus $v_t = 1/k$, which is the velocity corresponding to large t, may be called the terminal velocity. This is the velocity of 'touch down' or the velocity with which the particle terminates its journey.

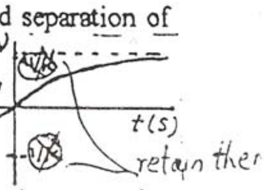
The same argument holds even if 'a' is expressed as $v\, dv/dx$. Thus $v\, dv/dx = g(1 - k^2v^2)$ and separation of variables followed by integration yields $v = \frac{1}{k}\sqrt{1 - e^{-2gk^2x}}$

As $x \to \infty, v \to \frac{1}{k}$

$† \int \frac{dv}{a^2 - v^2} = \frac{1}{2a} \ln\left(\frac{a+v}{a-v}\right)$

†† $\tanh u = \frac{e^u - e^{-u}}{e^u + e^{-u}}$

$u = gkt$



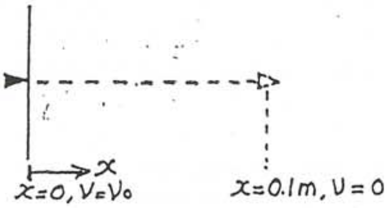
Prob.: A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_o = 360$ m/s and travels 100 mm before coming to rest. Assuming that the velocity of the projectile was defined by the relation $v = v_o - kx$ where v is expressed in m/s and x in meters, determine (a) the initial acceleration of the projectile, (b) the time required for the projectile to penetrate 94 mm into the resisting medium.

Given that $v = 360 - kx$

But $v = 0$, when x = 0.1 m. Thus k = 3600

$v = 360 - 3600x$

$v = 360\,(1-10x)$



Acceleration, $a = v\, dv/dx$

$= 360\,(1-10x)\,(-3600)$

When x=0 $a = -360(3600) = -1296000$ m/s^2

Therefore the initial acceleration of the projectile is **-1296 km/s^2**

To find the penetration time we need to find the value of t corresponding to x = 0.094m

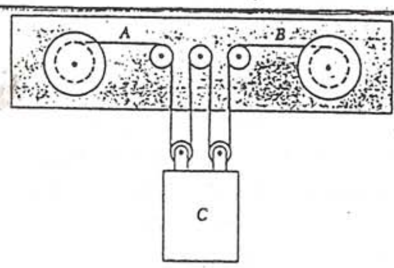
$dx/dt = 360\,(1-10x)$

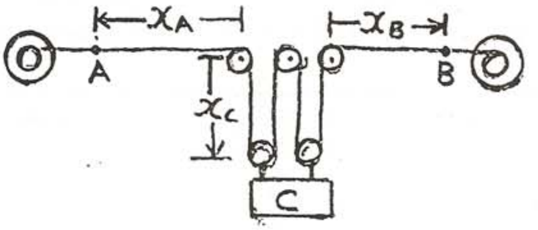
On integration, $\ln(1-10x)\,/\,-10 = 360t + C$

But at t = 0, x = 0. This implies that C = 0. Thus $x = 0.1(1-e^{-3600t})$

When x = 0.094 we find that $t = 7.82 \times 10^{-4}$ s; Therefore the penetration of 94 mm takes **782 μs**

Prob.: Under normal operating conditions, tape is transferred between the reels shown at a speed of 720 mm/s. At t=0, portion A of the tape is moving to the right at a speed of 600 mm/s and has a constant acceleration. If portion B of the tape has a constant speed of 720 mm/s and the speed of portion A reaches 720 mm/s at t = 6 s, determine (a) the acceleration and velocity of the compensator C at t = 4 s, (b) the distance through which C will have moved at t=6 s.





A and B are two fixed points on the left and right side portions of the tape. Even though A moves right and so does B, the length of the tape between A and B is constant.

With the datums fixed as in the figure, tape geometry dictates that $X_A+4X_C+X_B$ = constant length (i)

Hence $V_A+4V_C+V_B = 0$ (ii)

and $a_A + 4a_C + a_B = 0$ (iii)

At t = 0 $V_A = -600$ mm/s, $V_B = 720$ mm/s, Therefore **$V_C = -30$ mm/s**

At t = 6 $V_A = -720$ mm/s, $V_B = 720$ mm/s, Therefore $V_C = 0$

Thus velocity of C changes from -30 mm/s to zero in 6 seconds. However, its acceleration is a constant as seen from (iii) $a_A+4a_C + 0 = 0$, where $a_A = [-720-(-600)]/6$ (Constant acceleration of A) $= -20$ mm/s^2.

Thus $a_C = 5$mm/s^2

And $(V_C)_4 = (V_C)_0 + a_C(4)$

$= -30 + 5(4)$

$= -10$ mm/s

$(X_C)_6 = (V_C)_0 6 + \tfrac{1}{2} a_C 6^2$

$= -30(6) + \tfrac{1}{2}(5)(36) = -90$ mm

Interpreting the signs

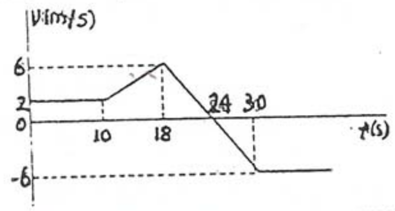
At t = 4s **$a_C = 5$ mm/s^2 (↓)**

$V_C = 10$ mm/s^2 (↑)

At t = 6s

$\Delta x_C = 90$ mm (↑)

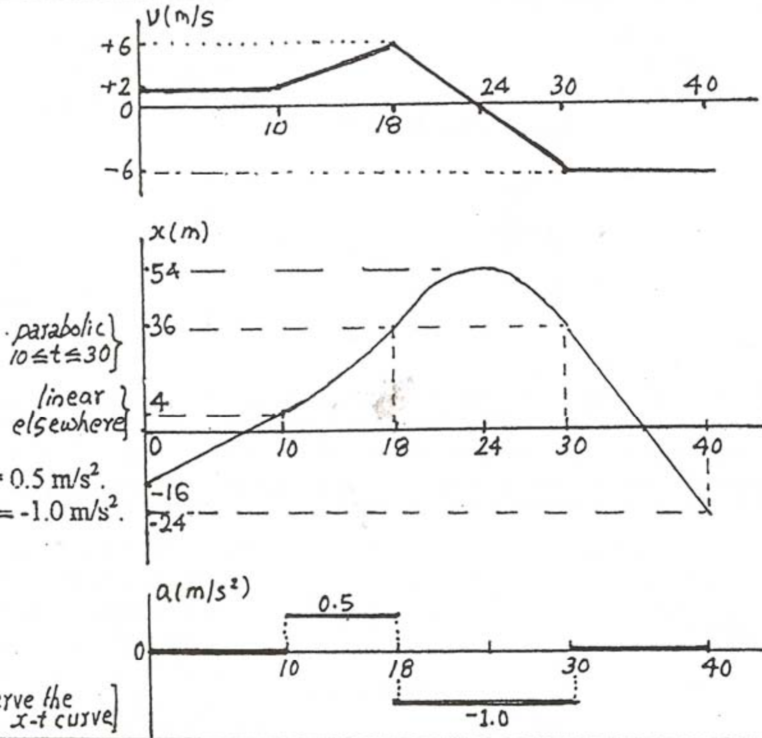
Prob.: A particle moves in a straight line with the velocity shown in the figure. If x = -16 m at $t = 0$, draw the a - t and x - t curves for $0 < t < 40$ s and determine *(a)* the maximum value of the position of the coordinate of the particle, *(b)* the value of t for which the particle is at a distance of 36 m from the origin, *(c)* the total distance traveled by the particle during the period $t = 0$ to $t = 30$ s, *(d)* the two values of t for which the particle passes through the origin.



$(Area)_{0\text{-}10} = 10 \times 2 = 20$ m
$(Area)_{10\text{-}18} = \frac{1}{2}8\,(2 + 6) = 32$ m
$(Area)_{18\text{-}24} = \frac{1}{2}(6)\,(6) = 18$ m
$(Area)_{24\text{-}30} = \frac{1}{2}(6)\,(-6) = -18$ m
$(Area)_{30\text{-}40} = 10\,(-6) = -60$ m

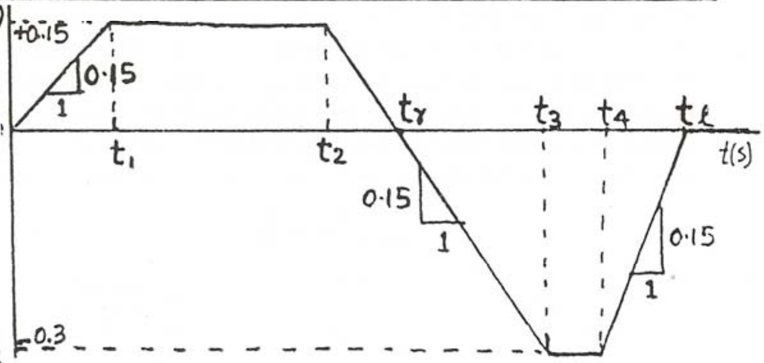
$x_0 = -16$ m (given)
$x_{10} = x_0 + (Area)_{0\text{-}10} = 4$ m.
$x_{18} = x_{10} + (Area)_{10\text{-}18} = 36$ m.
$x_{24} = x_{18} + (Area)_{18\text{-}24} = 54$ m.
$x_{30} = x_{24} + (Area)_{24\text{-}30} = 36$ m.
$x_{40} = x_{30} + (Area)_{30\text{-}40} = -16$ m.

$0 < t < 10$ s $a = 0$
$10\text{s} < t < 18$ s $a = (6-2) / (18-10) = 0.5$ m/s².
$18\text{s} < t < 30$ s $a = (-6-6) / (30-18) = -1.0$ m/s².
$30\text{s} < t < 40\text{s}$ $a = 0$
From the x – t curve $\mathbf{x_{max} = 54m}$
x = 36 m at **t = 18s and t = 30s**
In $0 < t < 30$s distance traveled is
[54 – (- 16)] + [54 – 36] or **88 m**.
x = 0 when **t = 8s and t = 36s.** [observe the x-t curve]



Prob.: During a finishing operation the bed of an industrial planer moves alternately 750 mm to the right and 750 mm to the left. The velocity of the bed is limited to a maximum value of 150 mm/s to the right and 300 mm/s to the left; the acceleration is successively equal to 150 mm/s² to the right, zero, 150 mm/s² to the left, zero, etc. Determine the time required for the bed to complete a full cycle, and draw the $v - t$ and $x - t$ curves.

$0 < t < t_r$ corresponds to the motion towards right whereas $t_r < t < t_\ell$ corresponds to that towards left.
From the v – t curve
$0.15/1 = 0.15/t_1 \Rightarrow t_1 = 1$s
Also $0.15/1 = 0.15/(t_r - t_2) \Rightarrow t_2 = t_r - 1$
$(Area)_{0\text{-}t_r} = 0.75\text{m} = \frac{1}{2}\,(0.15)[(t_2 - t_1) + t_r]$
or $0.75 = \frac{1}{2}(0.15)[t_r - 1 - 1 + t_r] \Rightarrow t_r = 6$s
Noting that $t_r = 6$s, $(t_3 - 6) / 0.3 = 1/0.15$



$\Rightarrow t_3 = 8$s and $(t_\ell - t_4)/0.3 = 1/0.15 \Rightarrow t_4 = t_\ell - 2$
$(Area)_{t_r\text{-}t_\ell} = 0.75\text{m} = \frac{1}{2}\,(0.3)[(t_4 - t_3) + (t_\ell - t_r)]$ or $0.75 = (1/2)(0.3)\,[t_\ell - 2 - 8 + t_\ell - 6] \Rightarrow t_\ell = 10.5$s
Thus the bed completes a full cycle in **10.5 s.**

Prob.: The motion of a particle is defined by the equations $x = t^2 - 8t + 7$ and $y = 0.5t^2 + 2t - 4$, where x and y are expressed in metres and t in seconds. Determine *(a)* the magnitude of the smallest velocity reached by the particle, *(b)* the corresponding time, position, and direction of the velocity.

$x = t^2 - 8t + 7$ $y = 0.5t^2 + 2t - 4$

$\therefore \dot{x} = 2t - 8$ $\quad\quad\quad\quad$ $\therefore \dot{y} = t + 2$

$V = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(2t-8)^2 + (t+2)^2}$

$\Rightarrow v = \sqrt{5t^2 - 28t + 68}$

V is smallest when $dv/dt = 0$ or $10t - 28 = 0$ i.e. when $t = 2.8s$

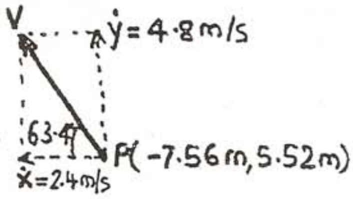
$V|_{t=2.8} = \sqrt{5(2.8)^2 - 28(2.8) + 68} = 5.367 \text{ m/s}$

$x|_{t=2.8} = (2.8)^2 - 8(2.8) + 7$
$= -7.56 \text{ m}$

$\dot{x}|_{t=2.8} = 2(2.8) - 8$
$= -2.4 \text{ m/s}$

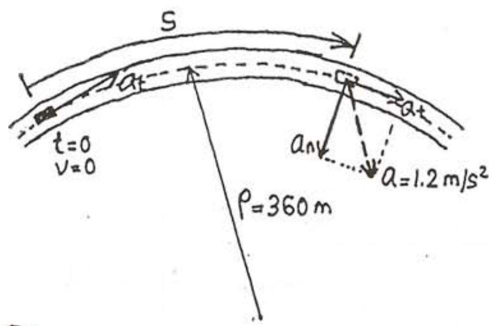
$y|_{t=2.8} = 0.5(2.8)^2 + 2(2.8) - 4$
$= 5.52 \text{ m}$

$\dot{y}|_{t=2.8} = 2.8 + 2$
$= 4.8 \text{ m/s}$



Smallest velocity is **5.367 m/s**. It occurs at t = **2.8s** when the particle is at **(-7.56m, 5.52m)** and the velocity is directed **63.4° north of west** (ie 63.4°)

Prob.: A monorail train starts from rest on a curve of radius 360 m and accelerates at the constant rate a_t. If the maximum total acceleration of the train must not exceed 1.2 m/s², determine *(a)* the shortest distance in which the train can reach a speed of 70 km/h, *(b)* the corresponding constant rate of acceleration a_t.



The train starts from rest at t = 0 (s = 0). The tangential acceleration a_t remains constant whereas a_n slowly increases as v increases. When the speed reaches 70 km/h (19.44 m/s)

$a_n = (19.44)^2/360 = 1.05 \text{ m/s}^2$

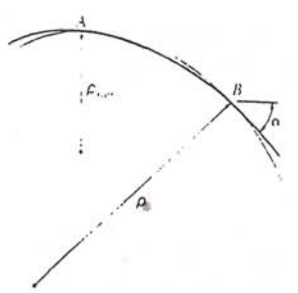
But $a_t = \sqrt{a^2 - a_n^2}$ where the total acceleration a cannot exceed 1.2 m/s²

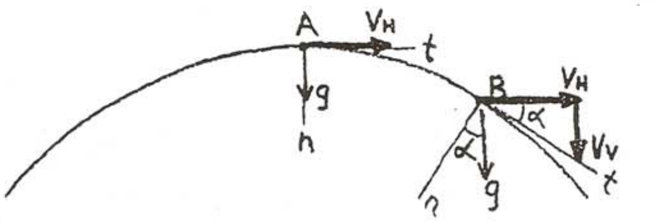
$\therefore a_t = \sqrt{1.2^2 - 1.05^2} \Rightarrow$ $\mathbf{a_t = 0.581 \text{ m/s}^2}$

The distance s, measured along the curve, covered by the train is found from $v^2 = u^2 + 2a_t s$

Where $v = 19.44$ m/s, $u = 0$, $a_t = 0.581 \text{ m/s}^2 \Rightarrow$ **s = 325.2m.**

Prob.: *(a)* Show that the radius of curvature of the trajectory of a projectile reaches its minimum value at the highest point *A* of the trajectory. *(b)* Denoting by α the angle formed by the trajectory and the horizontal at a given point *B*. Show that the radius of curvature of the trajectory at *B* is $\rho = \rho_{min}/\cos^3\alpha$.





Horizontal and vertical components of the velocity are denoted by V_H and V_V. The total velocity V is given by $V^2 = V_H^2 + V_V^2$

$= V_H^2 + V_H^2 \tan^2\alpha$ (as is obvious from the figure)

$= V_H^2 \sec^2\alpha$

The only acceleration of the projectile is that due to gravity. $a_n = g\cos\alpha$ and $\rho = v^2/a_n$

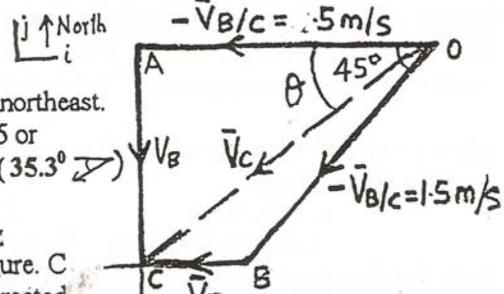
$$\rho = V_H^2 \sec^2 \alpha / g \cos \alpha \text{ or } \rho = V_H^2 / g \cos^3 \alpha \qquad \text{(i)}$$

In (i) when $\alpha = 0$ $\cos^3 \alpha$ is maximum and ρ is minimum But $\alpha = 0$ corresponds to point A, the highest on the trajectory. Thus $\rho_{min} = V_H^{\,2}/ g$ (ii)

From (i) and (ii) $\rho = \rho_{min}/\cos^3 \alpha$

Prob.: A small motorboat maintains a constant speed of 1.5 m/s relative to the water as it is maneuvering in a tidal current. When the boat is directed due east, it is observed from shore to move due south and when it is directed toward the northeast it is observed to move due west. Determine the speed and direction of the current.

Suffixes 'B','C', and 'S' correspond to boat, current and the shore respectively. Fixed frame is attached to the shore. $\bar{V}_{B/C} = \bar{V}_B - \bar{V}_C$, where $\bar{V}_B$ and $\bar{V}_C$ are the absolute velocities of the boat and the current. Note that $\bar{V}_{B/C} = 1.5$ m/s



Vector approach:

$\bar{V}_C = \bar{V}_B - \bar{V}_{B/C} \Rightarrow V_C = y\,j - 1.5\,i$ when boat is directed due east and $\bar{V}_C = xi - (1.5 \cos 45 i + 1.5 \sin 45 j)$ when the boat is directed northeast. Equating the coefficients of j in both the equations, $y = -1.5 \sin 45$ or -1.061 m/s $\therefore V_C = (-1.5\,i - 1.061\,j)$ m/s Thus $\mathbf{V_C = 1.837}$ **m/s** (35.3^0)

Geometry approach:

The vector equation can be geometrically interpreted as in the figure. C is the point obtained from the construction. When the boat is directed due east, $-V_{B/C}$ is given by the line OA and V_B is the unknown line AC directed south. When the boat is directed towards northeast, $-V_{B/C}$ is given by the line OB and V_B is the unknown line BC directed due west. This gives the point of intersection of AC and BC. Line OC is the vector corresponding to V_C.

$AC = 1.5 \sin 45$

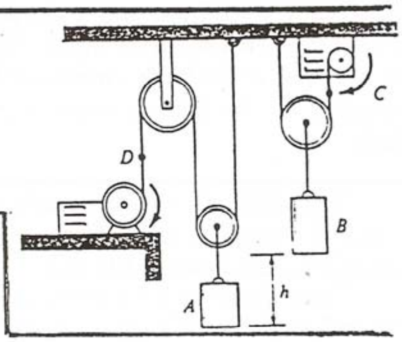
$OC = \sqrt{OA^2 + AC^2}$

$OC = \sqrt{1.5^2 + (1.5 \sin 45)^2} = 1.837$

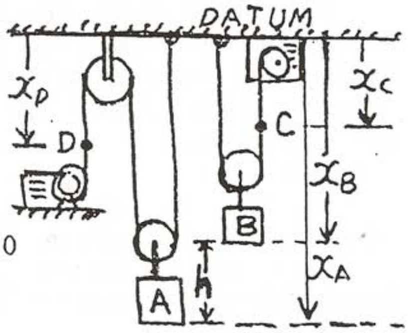
$\therefore V_C = 1.837$ m/s

$\theta = \tan^{-1}(1.5 \sin 45 / 1.5) = 35.3^\circ$

Prob.: The motor draws in the cable at *C* with a constant velocity of $V_C = 4$ m/s. The motor draws in the cable at *D* with constant acceleration of $a_D = 8$ m/s^2. If $v_D = 0$ and $h = 3$ m when $t = 0$, determine *(a)* the time needed for $h = 0$, and *(b)* the relative velocity of block *A* with respect to block *B* when this occurs.



There are two separate cables in the system. One end of each of the cables is fixed to the ceiling. Considering the cable geometry of both and noting that C and D are two fixed points on the cables, we get



$$2x_A + x_D = l_1 \qquad \text{(i)}$$

$$2x_B - x_C = l_2 \qquad \text{(ii)}$$

Differentiating (i) successively $2V_A + V_D = 0$ (iii)

$$2a_A + a_D = 0 \qquad \text{(iv)}$$

Differentiating (ii) successively $2V_B - V_C = 0$ (v)

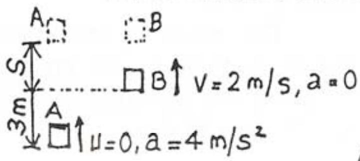
$$2a_B - a_C = 0 \qquad \text{(vi)}$$

Examining (iii) and (iv) at $t = 0$, $V_A = 0$ since $V_D = 0$

$a_A = -4$ m/s^2 since $a_D = 8$ m/s^2

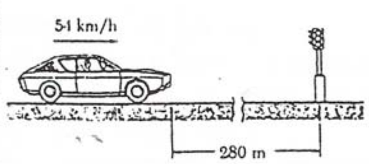
Examining (v) and (vi) we get, $V_B = -2$ m/s since $V_C = -4$ m/s, $a_B = a_C = 0$

Interpreting the signs of velocity and acceleration of A and B we get a clear picture of the motion. Block A starts with zero velocity and moves upwards at 4 m/s^2 acceleration whereas block B moves up at a constant speed of 2 m/s. When A and B are at same level, h = 0.



For A, $(3 + S) = 0 + \frac{1}{2}(4)\,t^2$ and for B, $S = 2t + 0$ where t is time of travel. Solving for t we get $\mathbf{t = 1.823\ s}$. At $t = 1.823$ s $V_A = 0 + 4(1.822)$ m/s and $V_B = 2$ m/s. But $V_{A/B} = V_A - V_B \Rightarrow V_{A/B} = \mathbf{5.291}$ **m/s** (↑)

Prob.: A motorist is traveling at 54 km/h when she observes that a traffic signal 280 m ahead of her turns red. She knows that the signal is timed to stay red for 28 s. What should she do to pass the signal at 54 km/h just as it turns green again? Draw the $v-t$ curve, selecting the solution which calls for the smallest possible deceleration and acceleration, and determine *(a)* the deceleration and acceleration in m/s², *(b)* the minimum speed reached in km/h.
If the acceleration of the automobile cannot exceed 0.6 m/s², what would be the minimum speed reached. Find the corresponding deceleration.

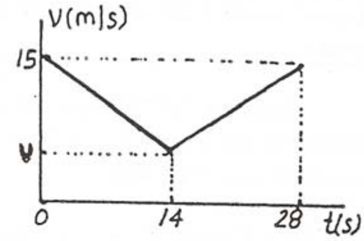


The distance to be covered is 280 m in 28 s. Initial and final velocities are 15 m/s (54 km/h)

The figure shows the v – t curve calling for the minimum acceleration and deceleration the automobile. Area under the v – t curve is the area of the rectangular block (28 x 15) minus the area of the triangle ½(28) (15 – v). This equals 280 m. *Therefore*

$280 = 28(15) - \frac{1}{2}(28)(15-V) \Rightarrow$ **V = 5 m/s** or **18 km/h**

† acceleration and deceleration = (15-5) /14 = **0.714 m/s²**

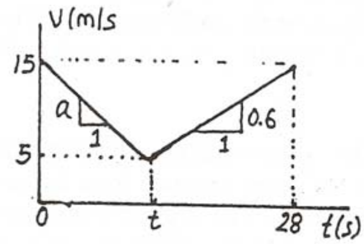


When acceleration cannot exceed 0.6 m/s² the slopes of the legs of the v – t curve are different.

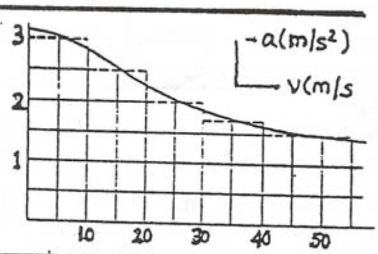
$$\left.\begin{array}{l}(15-5)/t = a \\ (15-5)/(28-t) = 0.6\end{array}\right\} \begin{array}{l}\Rightarrow t = 11.35 \\ \text{and } a = 0.822 \text{ m/s}^2\end{array}$$

Thus the minimum speed reached is **18 km/h** (as in the previous case) and the deceleration is **0.822 m/s²**

† Smallest acceleration and deceleration occur when slopes of both legs of the v – t curve are equal.



Prob.: The maximum possible deceleration of a passenger train under emergency conditions was determined experimentally; the results are shown (solid curve) in the figure. If the brakes are applied when the train is traveling at 90 km/h, determine by approximate means *(a)* the time required for the train to come to rest, *(b)* the distance traveled in that time.

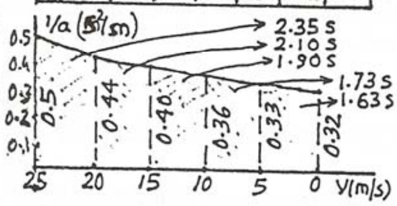


Since (1/a) dv = dt, if a curve of 1/a versus v is plotted and the area under the curve is taken from v = 25 m/s (or 90 km/h) to v = 0 we directly get time required to stop.

V (m/s)	25	20	15	10	5	0
-a (m/s²)	2	2.25	2.5	2.8	3.0	3.15
$-\frac{1}{a}$ (s²/m)	0.50	0.44	0.40	0.36	0.33	0.32

From the given a – v curve the values of a and hence 1/a are found for various values of v (v = 25 m/s to v= 0 at 5 m/s travels)

Note how the various areas are approximated by the trapezoidal rule and the corresponding time intervals have been calculated. For example, in the 15 m/s to 10 m/s interval 1/a = (0.4+0.36)/2 = 0.38 s²/m

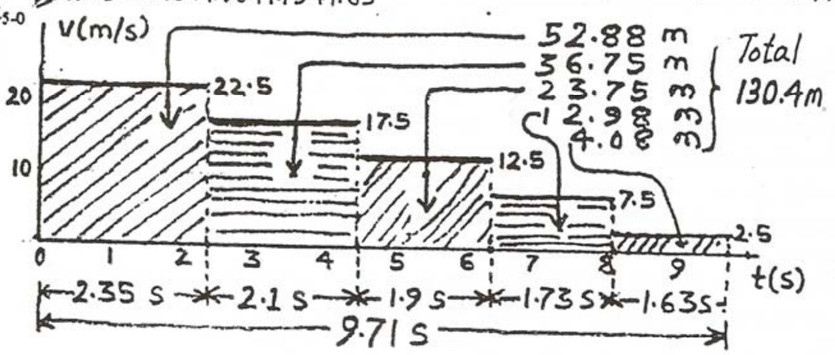


∴ Area = 0.38 s²/m x 5 m/s = 1.9 s

$\Delta t = \Delta t_{25-20} + \Delta t_{20-15} + \ldots\ldots\ldots \Delta t_{5-0}$ = 2.35 + 2.10 + 1.90 + 1.73 + 1.63
= **9.71s**

To find the distance covered by the train we plot the v – t curve and take the area bound by the curve between t = 0 and t = 9.71 s.

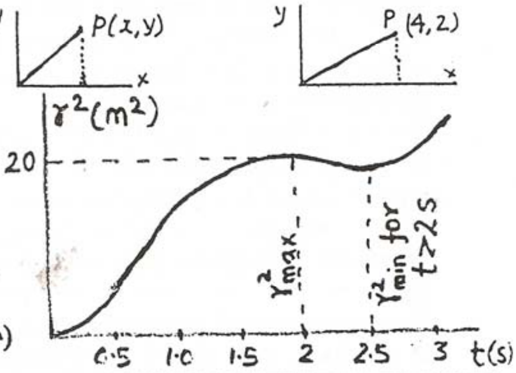
The speed in each time interval is taken as the average in the interval. Distance traveled in 9.71 s is **130.4 m.**



Prob.: A particle has component velocities $V_x = (5 - 3t)$ m/s, $V_y = t$ m/s. At $t = 0$, $x = y = 0$. Find equations for x and y and thus obtain an expression for the distance r of the particle from the origin.

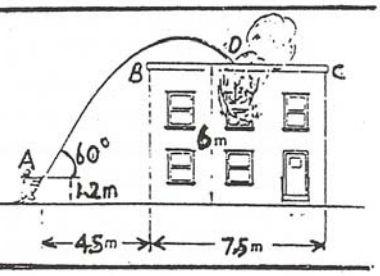
(a) Find the farthest distance the particle reaches from the origin in the time period 0 to 2.6 s.
(b) At what time does this occur?
(c) State the significance of the instant $t = 2.5$ s.
(d) Find the absolute velocity and acceleration of the particle when $t = 2.0$ s.

$V_x = 5 - 3t$. On integration and substitution $x = 0$ at $t = 0$, we get $x = 5t - 1.5t^2$. Similarly on integrating $V_y = t$ and substituting $y = 0$ at $t = 0$, we get $y = 0.5\,t^2$. Distance from the origin (r) is given by $r = \sqrt{x^2+y^2}$
$= \sqrt{(5t - 1.5t^2) + 0.5t^2}$ or $r^2 = 2.5\,(t^4 - 6t^3 + 10t^2)$ (i)
r is maximum when $d/dt(r^2) = 0$
$d/dt(r^2) = 2.5\,(4t^3 - 18t^2 + 20t) = 0 \Rightarrow t = 0, 2, 2.5$
$d^2/dt^2(r^2) = 2.5\,(12t^2 - 36t + 20)$. At $t = 0$ and $t = 2.5$ the quantity $d^2/dt^2(r^2)$ is positive, whereas at $t = 2$ s it is negative. Therefore, **at t = 2 s particle is farthest from the origin.** The distance is obtained from (i)
$\mathbf{r_{max} = 4.472}$ **m**
It can be seen that $x|_{t=2} = 4$ m & $y|_{t=2} = 2$ m
At t = 2.5 s the particle is closest to the origin for t > 2s.
The figure given is only a plot of r^2 verses t. At t = 2 s, $r^2 = 20$ m^2 and at t = 2.5 s $r^2 = 19.53$ m^2



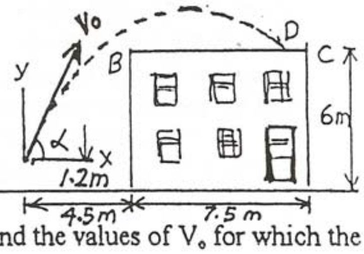
$V_x = (5 - 3t)$ m/s $a_x = -3$ m/s^2
$V_y = t$ m/s $a_y = 1$ m/s^2
At t = 2 s, $V_x = -1$ m/s   At t = 2 s $a_x = -3$ m/s^2
$V_y = 2$ m/s $a_y = 1$ m/s^2
$\Rightarrow$ **V = 2.23 m/s** (63.4°) $\Rightarrow$ **a = 3.162 m/s^2** (18.4°)

Prob.: A nozzle at A discharges water with an initial velocity of 12 m/s at an angle of 60° with the horizontal. Determine where the stream of water strikes the roof. Check that the stream will clear the edge of the roof. Also, determine the range of values of the initial velocity for which the water fall on the roof.



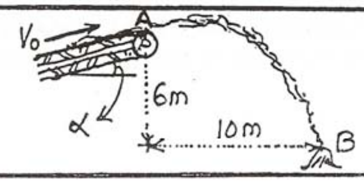
Let the stream strike the roof at D. Its coordinates are (x, 4.8 m), $\alpha = 60^0$, $V_0 = 12$ m/s. Substituting the coordinates of D into the trajectory equation $(6 - 1.2) = x\tan 60 - g\,x^2 / [2(12)^2 \cos^2 60]$

Solving the quadratic equation in x, x = 4.083 m, 8.625 m x = 4.083 m is discarded since the jet has not reach the building. But x = 8.625 m is a genuine solution. The jet thus strikes the roof at a distance of 8.625 m – 4.5 m i.e. **4.125 m away from B**. To check whether the stream clears the edge B, we only need to show that y > 4.8 m when x = 4.5. That is $y = 4.5 \tan 60 - g(4.5)^2/2(12)^2 \cos^2 60 = 5.04$ m (> 4.8 m) so that stream does clear the edge.

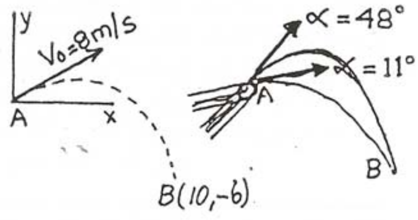


To find the range of V_0 for which the stream strikes the roof we need to find the values of V_o for which the stream strikes B and C.
For B, $4.8 = 4.5 \tan 60 - g(4.5)^2 / [2v^2_{min} \cos^2 60] \Rightarrow V_{min}$ 11.52 m/s
For C, $4.8 = 12 \tan 60 - g(12)^2 / [2v^2_{max} \cos^2 60] \Rightarrow V_{max}$ 13.29 m/s
To check whether the stream clears the roof for all velocities between V_{min} & V_{max}. We verify that 'y' corresponding to all such 'V's are greater than 4.8 m at x = 4.5 m i.e. $4.5 \tan 60 - g(2.5)^2/2v^2 \cos^2 60 > 4.8$ m
Range of V_o is thus **11.52 m/s ≤ V_o ≤ 13.29 m/s.**

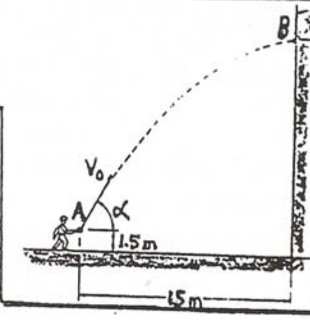
Prob. : If the conveyor belt moves at the constant speed Vo = 8 m/s determine the angle α for which the sand is deposited on the stock-pile at B.



Point B. where the sand gets deposited, lies on the trajectory. Its coordinates are(10 m,-6 m). These are substituted into the trajectory equation to get $-6 = 10\tan\alpha - g\,(10)^2/[2(8)^2\cos^2\alpha]$
Expressing $\cos^2\alpha$ as $1/(1+\tan^2\alpha)$ we get a quadratic equation in $\tan\alpha$ which is $7.66\tan^2\alpha - 10\tan\alpha + 1.66 = 0$
Solving this for $\tan\alpha$ and then for α, $\boldsymbol{\alpha = 11^\circ, 48^\circ}$
Figure shows the two angles for which the sand gets deposited.



Prob.: A player throws a ball with an initial velocity Vo of 18 m/s from point A. (a) Determine the maximum height h at which the ball can strike the wall, (b) the corresponding angle α.



It is to be borne in mind that every feasible angle of projection α is associated with a particular striking point on the wall. Out of all such values of α, one is associated with the highest point. Our aim is to find that point's location. This is done by first framing an equation that relates α and h. Substituting the coordinates of the striking point B into the equation of the trajectory.

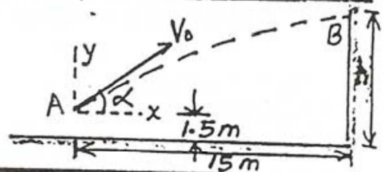
$(h - 1.5) = 15\tan\alpha - g(15)^2/2(18)^2\cos^2\alpha$ (i)

If h is to be maximum $dh/d\alpha = 0$

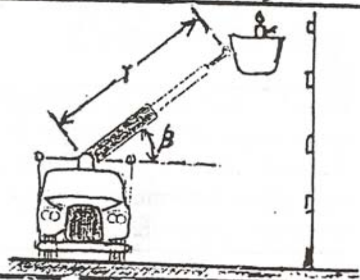
$dh/d\alpha = 15\sec^2\alpha - 3.406\,(2\sec\alpha\sec\alpha\tan\alpha) = 0$

$\Rightarrow \tan\alpha = 15/[2(3.406)]$ or $\boldsymbol{\alpha = 65.6^\circ}$

When this value of α is substituted into (i) we get h_{max}. Thus, $\mathbf{h_{max} = 14.6\ m}$.



Prob. : A fire truck has a telescoping boom holding a fireman as shown in the diagram. At time *t*, the boom is extending at the rate of 0.6 m/s and increasing its rate of extension at 0.3 m/s^2. Also, at time *t*, $r = 10$ m and $\beta = 30^0$. If a velocity component of the man of 3.3 m/s vertically is desired, what should $\dot\beta$ be? Also, if a vertical acceleration component of the man of 1.7 m/s^2 is desired, what should $\ddot\beta$ be?



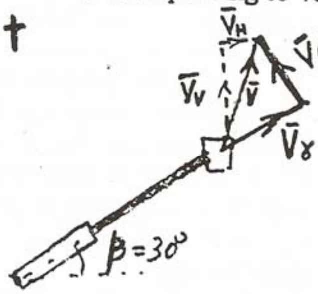
At the instant given
$r = 10$ m $\beta = 30^0$ $\dot r = 0.6$ m/s $\ddot r = 0.3$m/s^2. $\dot\beta$ and $\ddot\beta$ are to be found corresponding to vertical velocity of 3.3 m/s and acceleration of 1.7 m/s^2

From the adjoining figure it is clear that $\bar V_r + \bar V_\beta = \bar V_V + \bar V_H = \bar V$ and hence
$V_V = V_r\sin\beta + V_\beta\cos\beta$ (i)
Note that V_r and V_β are the radial and transverse velocities whereas V_V and V_H are the vertical and horizontal ones.From (i)

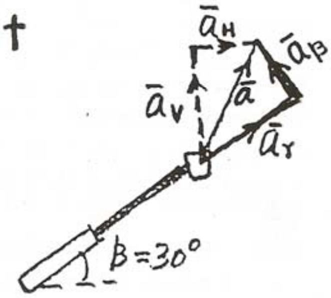
$V_V = \dot r\sin 30 + r\dot\beta\cos 30$ (ii)

But $V_V = 3.3$ m/s, $\dot r = 0.6$ m/s, $r = 10$ m and hence from (ii)

$\boldsymbol{\dot\beta = 0.3464}$ **rad/s**

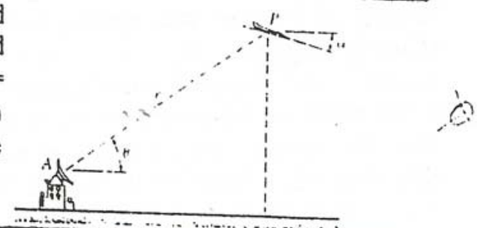


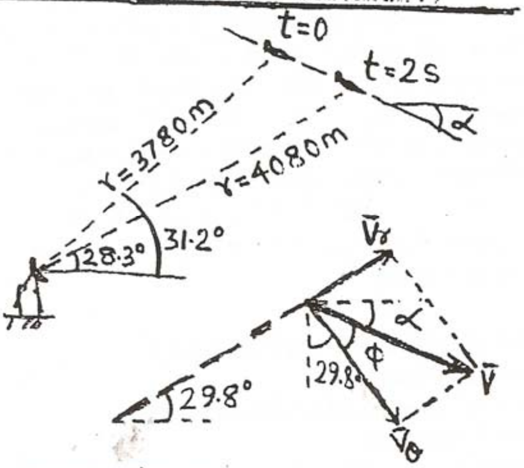
$a_V = a_r\sin\beta + a_\theta\cos\beta$
Required $a_V = 1.7$ m/s^2 and it is known that
$a_r = \ddot r - r\dot\beta^2$, $a_\theta = r\ddot\beta + 2\dot r\dot\beta$
$\therefore 1.7 = [0.3 - 10(0.3464)^2]\sin 30 + [10\ddot\beta + 2(0.6)(0.3464)]\cos 30$
$\Rightarrow \boldsymbol{\ddot\beta = 0.2068}$ **rad/s²**



Idea conveyed is only $\bar V = \bar V_H + \bar V_V = \bar V_r + \bar V_\beta$ *and* $\bar a = \bar a_H \pm \bar a_V = \bar a_r + \bar a_\theta$
Figures may not be showing the correct directions of $\bar V_H$ *and* $\bar a_H$

Prob.: An airplane passes over a radar tracking station at A and continues to fly due east. When the airplane is at P, the distance and angle of elevation of the plane are, respectively, $r = 3780$ m and $\theta = 31.2^0$ Two seconds later the radar station sights the plane at $r = 4080$ m and $\theta = 28.3^0$. Determine approximately the speed and the angle of dive α of the plane during the 2 s interval.





We work in the r - θ system characterized by the radar parameters.

$\bar{v}_r = \dot{r}.\bar{e}_r$, $\bar{v}_\theta = r\dot{\theta}\,\bar{e}_\theta$ where $\bar{e}_r$ and $\bar{e}_\theta$ are unit vectors in the radial and transverse directions. $\dot{r}$ and $\dot{\theta}$ can only be approximated in the given 2s interval and so can r and θ be.

$\dot{r} = (4080-3780) / 2 = 150$ m/s,

$\dot{\theta} = [(28.3^0 - 31.2^0)\pi/180] / 2 = -0.0253$ rad/s

$r = (3780+4080) / 2 = 3930$ m

$\theta = (31.2^0 + 28.3^0) / 2 = 29.8^0$

$v_r = \dot{r} = 150$ m/s and $v_\theta = r\dot{\theta} = 3930\,(-0.0253) = -99.43$ m/s

$v \approx \sqrt{v_r^2 + v_\theta^2} \Rightarrow$ **speed ≈ 648 km/h**

From the figure, the angle of dive can be found as

$\alpha = 90^0 - 29.8^0 - \tan^{-1}(v_r/v_\theta) = 90^0 - 29.8^0 - 56.5^0 = 3.7^0$

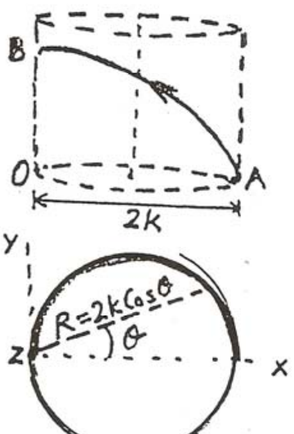
Angle of dive ≈ 3.7°

Prob.: The three dimensional motion of a particle is defined by the relations R = 2k cos t, θ = t, and z = pt, Determine (a) the path of the particle, (b) the magnitudes of the velocity and acceleration of any time t, (c) the radius of curvature of the path at any time t.

Motion is defined in the cylindrical coordinates. Path of the particle is as given in the figure from A to B. Vertical axis is at R = k, θ = 0.

Path is sketched for $0 \le t \le \pi/2$

At time		
t	0	π/2
R	2k	0
θ	0	π/2
z	0	p π/2



If R - θ plane is converted into x – y plane and z axis is retained,

$x = R\cos\theta$, $y = R\sin\theta$, $z = pt$

or $x = 2k\cos^2 t$, $y = k\sin 2t$, $z = pt$ (i)

Differentiating (i)

$\dot{x} = -2k\sin 2t$, $\dot{y} = 2k\cos 2t$, $\dot{z} = p$ (ii)

Differentiating (ii)

$\ddot{x} = -4k\cos 2t$, $\ddot{y} = -4k\sin 2t$, $\ddot{z} = 0$ (iii)

From (ii) and (iii) and noting that

$v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ and $a = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$

we get $\mathbf{v = \sqrt{4k^2 + p^2}}$ and $\mathbf{a = 4k}$

We know that $a_t = \bar{e}_t \cdot \bar{a}$ where a_t is the tangential acceleration component and $\bar{e}_t$ is a unit vector in the tangential direction. e_t is found by dividing the velocity vector by its magnitude Thus

$e_t = \dfrac{(-2k\sin 2t\, i + 2k\cos 2t\, j + p\, k)}{\sqrt{4k^2 + p^2}}$ and $a = -4k\cos 2t\, i - 4\, k\, 2\sin 2t\, j$

$a_t = \bar{e}_t \cdot \bar{a}$ gives $a_t = 0$. The total acceleration is therefore a_n only.

$\rho = v^2 / a_n$ simplifies to $\rho = (4k^2 + p^2) / 4k$ or $\rho = \mathbf{k + (p^2 / 4k)}$

Prob.: A pilot boat leaves port to intercept a tanker which is sailing at 30 km/h on a straight course whose nearest point is 5 km from the port. At the instant the pilot boat leaves port the tanker is 8 km away.(a) At what minimum speed must the pilot boat sail in order to intercept the tanker? (b) If the pilot boat sails at 20 km/h for how long is the tanker in a position to be intercept by the pilot boat.

For the boat (B) to intercept the tanker (T), the relative velocity of the boat with respect to tanker must be directed along the 8 km path. $\bar{V}_B = \bar{V}_T + \bar{V}_{B/T} \Rightarrow V_B = 30\text{km/h} (\downarrow) + V_{B/T} (\measuredangle 51.32^0)$ Figure shows the pictorial representation of the vector equation. Minimum speed of the boat corresponds to when its absolute velocity vector is perpendicular to the 8km path. Thus $AE = 30\cos\theta = 18.75$km/h. Thus

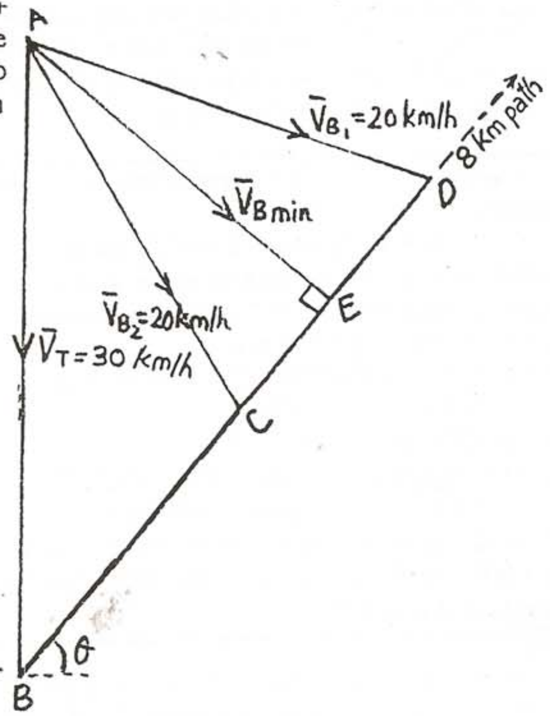
V_{min} **= 18.75 km/h.**

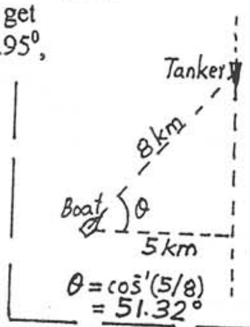
When the boat sails at 20km/h, the 8km path is cut in two point C and D. D and C correspond to first and last possible interception respectively since BD and BC respectively represent the relative velocity of boat with respect to the tanker. From trigonometry we can get

$\angle ACE = \angle ADE - 69.3^0$, $\angle BAC = 30.95^0$,

$\angle CAD = 40.74^0$, $\angle ABC = 38.68^0$

$\angle ACB = 110.4^0$.





$BC = AC (\sin\angle BAC / \sin \angle ABC) \Rightarrow$

BC = 16.46 km/h.

$CD = AC (\sin\angle CAD / \sin \angle ACD) \Rightarrow$

CD = 13.92 km/h.

For the first possible interception,

$t_1 = 8\text{km} / V_{B/T} = 8 / BD = 8 / (BC+CD) = 8 / (16.46 + 13.92)$.

For the last interception, $t_2 = 8$ km / BC = 8 / 16.46.

The 'risk span' is obviously $t_2 - t_1$. $t_2 - t_1 = [8 / (16.46 + 13.92)] - (8 / 16.46) = 2.23$ h. or 13.38 min.

The tanker is therefore in a position to be intercepted by the pilot boat for 13.38 minutes.

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