Numerical Methods

Notes by-

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Notes 1

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Section F

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Numerical Methods:-

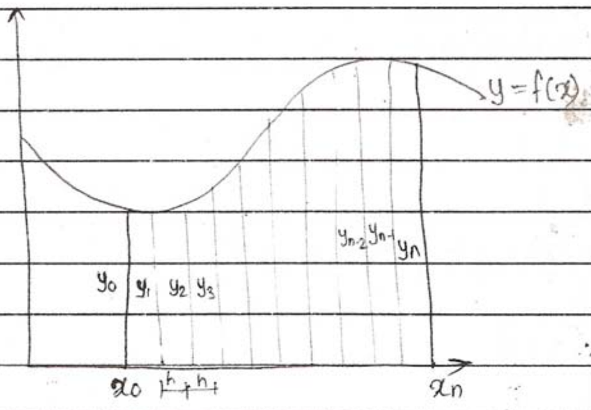
1] Finding Area By Simpsons Rule, / Trapezoidal Methods.

If we want to find c/s ~~of~~ area of river bed, since river bed is a natural formation, it cannot possess analytical expression $y = f(x)$; & hence we cannot evaluate $A = \int_a^b f(x)\,dx$.

Here the concept of numerical integratⁿ comes to our help.

a) Trapezoidal Rule:-

- Newton - cotes gives first formula to find integration.
- Trapezoidal rule is first amoung them & is a "Closed form of formula"



Area between x_0 & x_n below a curve $y = f(x)$ is calculated by dividing the area in to number of sections & or strips having same width.

$\therefore I = \int_{x_0}^{x_n} y \cdot dx$ ... Basic Defⁿ of Integratⁿ

$$\boxed{\therefore I = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + y_3 + \cdots + y_{n-1})\right]}$$

Wher n may be odd or even.

- Algorithm:-

Step 1: Read the No. of intervals, say 'n'

Step 2: Read the limits of integration, say x_0, x_n

Step 3: Compute the stepsize, $h = \frac{x_n - x_0}{n}$

Step 4: Compute the 'y' for all the corresponding 'x' values. i.e. $y_i = f(x_0 + i \cdot h)$ for $[i = 1, 2, \ldots, n-1]$

Step 5: SUM = 0 ; $i = 1 + i$ i.e. $i = 2$ (counter) step 6: $SUM = SUM + 2y_i \cdot i = i+1$

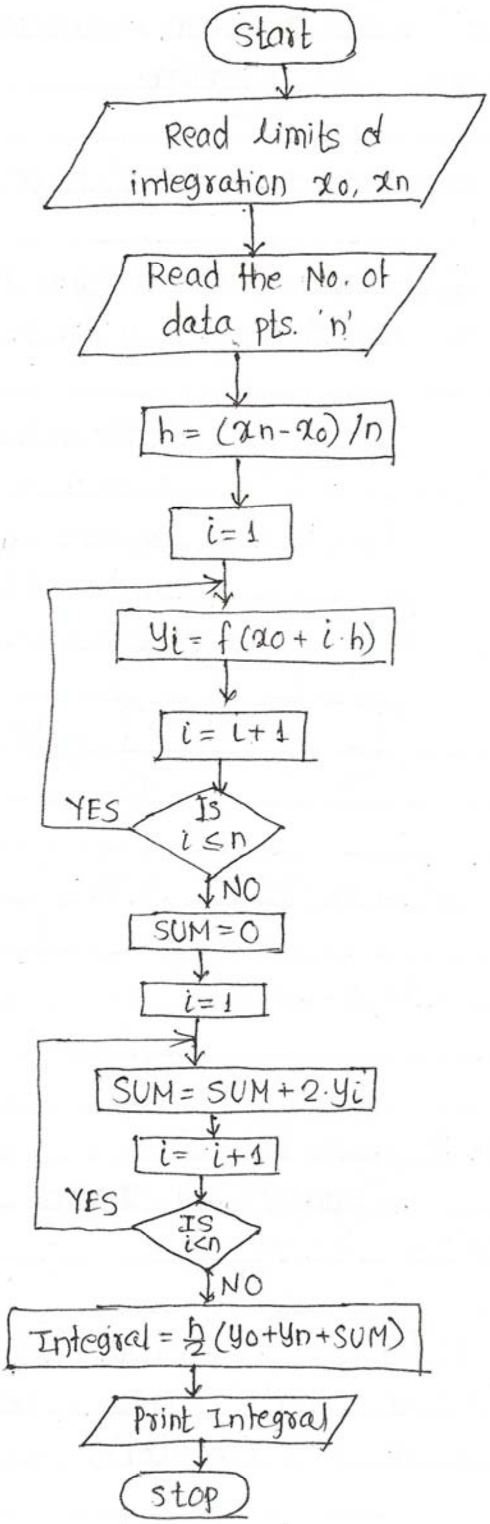
$i\ (i \le n)$ then go to step '6'

step 7: Integral $= \frac{h}{2}\,[y_0 + y_n + SUM]$

step 8: Display integral

step 9: Stop.

Flow Chart:-



Pro: 1 $\int_0^{0.8} e^{x^2}\,dx$: Use the trapezium Rule by choosing $h = 0.1$

$\frac{N}{2}$

Soln.:- Let $y = f(x) = e^{x^2}$

$h = 0.1$

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
$y = e^{x^2}$	1	1.0101	1.0408	1.0942	1.1735	1.284	1.4333	1.6323	1.8965

$\therefore I = \int_0^{0.8} e^{x^2}\,dx = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$

$= \frac{0.1}{2}\left[1 + 1.8965 + 2(1.0101 + 1.0408 + 1.0942 + 1.1735 + 1.284 + 1.333 + 1.6323)\right]$

$\boxed{\cancel{I = 1.0817}}$ $\boxed{I = 1.01165}$ ... approx.

Pro: 2] Evaluate, $I = \int_0^1 \frac{dx}{1+x}$ by trapezoidal rule with $h = 0.125$.

Soln:- $I = \int_{x_0}^{x_n} f(x)\cdot dx = \int_0^1 \frac{dx}{1+x}$; $h = 0.125$.

x	0	0.125	0.25	0.375	0.5	0.625	0.75	0.875	1.0
$y = \frac{1}{1+x}$	1	0.8889	0.8	0.7273	0.6667	0.6154	0.5714	0.5333	0.5

$I = \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})\right]$

$= \frac{0.125}{2}\left[1 + 0.5 + 2(0.8889 + 0.8 + 0.7273 + 0.6667 + 0.6154 + 0.5714 + 0.5333)\right]$

$\therefore$ $\cancel{I = 0.6275}$ $\boxed{I = 0.694125}$ ... approx.

Pro: 3] $I = \int_3^4 \frac{3x}{2+\sin^2 x}\,dx.$

Degree

x	3	3.125	3.25	3.375	3.5	3.625	3.75	3.875	4.0
$y = \frac{3x}{2+\sin^2 x}$	4.4938	4.6805	4.8672	5.0537	5.2402	5.4267	5.6130	5.7993	5.9854

WRONG

Mode: Rad

x	3	3.125	3.25	3.375	3.5	3.625	3.75	3.875	4
$y = \frac{3x}{2+\sin^2 x}$	4.4556	4.6869	4.8466	4.9306	4.9457	4.9074	4.8352	4.7486	4.6643

~~Rad~~

$\therefore I = \frac{0.125}{2}\left[4.4938 + 5.9854 + 2(4.6805 + 4.8672 + 5.0537 + 5.2402 + 5.4267 + 5.613 + 5.7993)\right]$

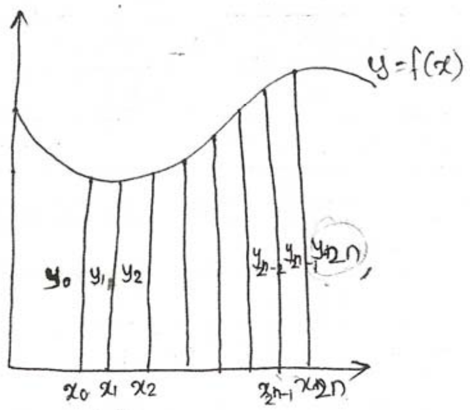
$\boxed{\cancel{I = 5.9882 \text{ Degree}}}$ 5.24

$\boxed{I = 5.24 \text{ Degree}}$ ... approx.

$$I = \frac{0.125}{2}\left[4.4556 + 4.6643 + 2(4.6869 + 4.8466 + 4.9306 + 4.9457 + 4.9074 + 4.8352 + 4.7486)\right]$$

$\therefore \boxed{I = 4.8076 \text{ (rad)}}$ $\boxed{4.8112}$ ... approx.

b) Simpson's 1/3 Rule :- (n should be odd)



First we integrate $\int_{x_0}^{x_2} y \cdot dx$ by considering a double strip under the curve.

Generalized form :-

$$I = \int_{x_0}^{x_{2n}} y \cdot dx$$

$$\therefore I = \frac{h}{3}\left[(y_0 + y_{2n}) + 4(y_1 + y_3 + y_5 + \cdots + y_{2n-1}) + 2(y_2 + y_4 + \cdots + y_{2n-2})\right]$$

where, y_0 & y_{2n} are end ordinates.

y_1, y_3, y_{2n-1}, are second, fourth ... or even ordinate

$y_2, y_4, \ldots, y_{2n-2}$ are third, fifth ... or odd ordinate.

$\therefore I = \frac{h}{3}$ [(sum of end ordinates) + 4(sum of even ordi.) + 2(sum of odd ordinates)]

* Algorithm for Simpson's 1/3 Rule :-

Step 1 : Read total No. of data points, say 'n'.

Step 2 : Read the limits of integration say x_0, x_n.

Step 3 : Compute the stepsize, $h = \frac{x_n - x_0}{n}$

Step 4 : Compute the 'y' for all the corresponding 'x' values.

i.e. $y_i = f(x_0 + i \cdot h)$ for $(i = 1, 2, \ldots, n-1)$

Step 5 : Obtain sum of odd data points,

i.e. Sum ⇒ odd $= y_1 + y_3 + y_5 + \cdots$

Step 6 : obtain sum of even data points,

i.e. sum of even $= y_2 + y_4 + y_6 + \cdots$

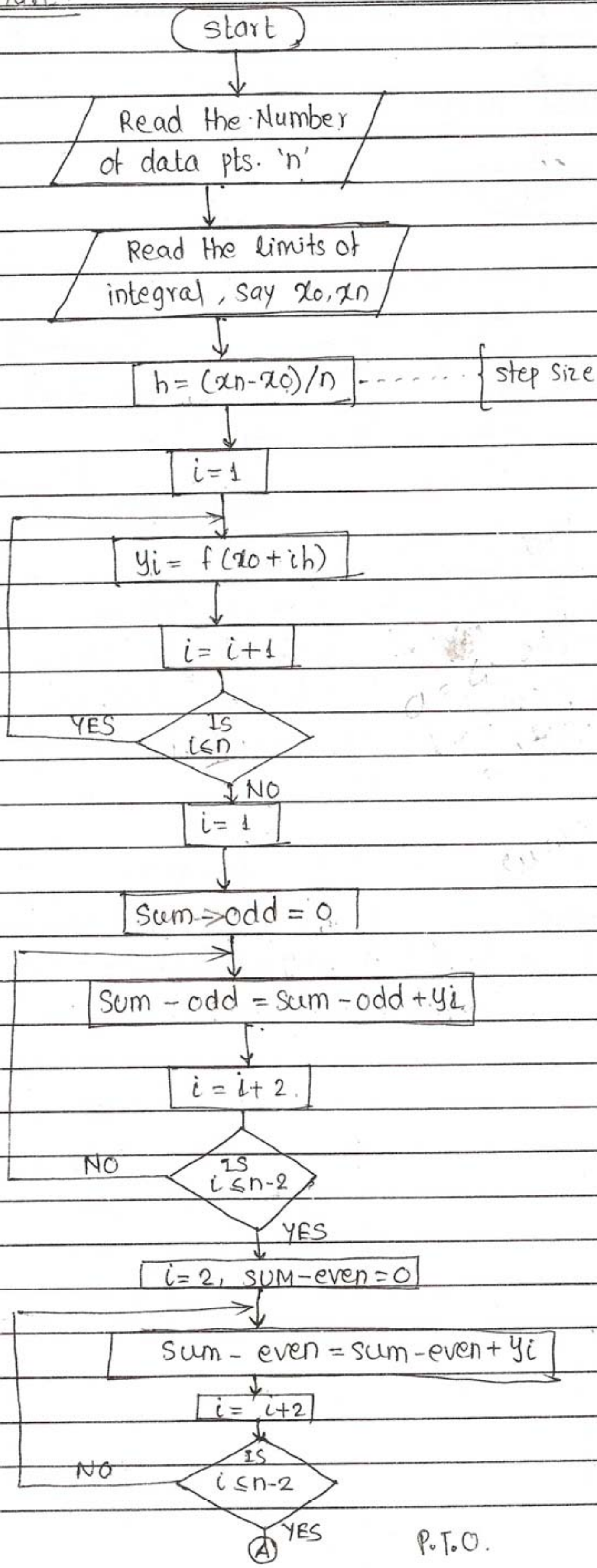
Step 7 : Calculate integral value as,

$I = \frac{h}{3}$ [y_0 + 4 (sum of odd) + 2 (sum of even) + y_n]

Step 8 :- Display integral

Step 9 :- stop.

Flow chart :-



P.T.O.

(A)

Integral = $\frac{h}{2}$ [$y_0 + y_n$ + 4(sum odd) + 2(sum even)]

Print Integral

Stop

Decision Tables: -

$wl = Ra(l+a)$

$Ra = \frac{w.(la-a)}{l+a}$

* Decision tables are used to define clearly and concisely the word stmt of a problem in tabular form
They can prove to be a powerful tool for defining complex problem logic.

Pseudocode is another programming analysis tool that is used for planning program logic. imitation of actual computer instructions.

* Simpson's 3/8th Rule:-

To use Simpsons 3/8th rule, we must have $(3n+1)$ ordinates.
Let $(x_0, y_0), (x_1, y_1), \dots (x_{3n}, y_{3n})$ be $(3n+1)$ number of pts.
& h is the common diff. betⁿ the arguments, then,

$$\boxed{A = \int_{x_0}^{x_{3n}} y \cdot dx = \frac{3h}{8}\left[(y_0+y_{3n}) + 3(y_1+y_2+y_4+y_5+y_7+\cdots) + 2(y_3+y_6+y_9+\cdots)\right]}$$

i.e. $I = \frac{3h}{8}$ [(sum of end ordinates) + 3(sum of ordinates with suffixes not multiple of 3) + 2(sum of ordinates with suffixes multiple of 3)]

Pro: Evaluate $\log_e 7$ by simpson's (i) $\frac{1}{3}^{rd}$ & (ii) $\frac{3}{8}^{th}$ Rule

Let $I = \int_0^6 \frac{1}{1+x} \cdot dx = [\log(1+x)]_0^6$

$= \log_e(1+6) - \log_e 1$

$= \log_e 7$

$\therefore \log_e 7 = \int_0^6 \frac{1}{1+x} dx = \int_0^6 y \cdot dx$

$\therefore x$	0	1	2	3	4	5	6	7
$y = \frac{1}{1+x}$	1	0.5	0.3333	0.25	0.2	0.1666~~7~~	0.1429	
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	

(i) Simpson's 1/3rd Rule,

$I = \frac{h}{3}[y_0 + y_6 + 4(y_1+y_3+y_5) + 2(y_2+y_4+\cancel{y_6})]$

$= \frac{1}{3}[\cancel{0\ 1+0.5+4(}\ 1+0.1429 + 4(0.5+0.25+0.1667) + 2(0.3333+0.2+\cancel{0.14})$

$\therefore \boxed{I = 1.958767}$ ... approx.

(ii) Simpsons 3/8th Rule,

$I = \frac{3}{8}h[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)]$

$\therefore \boxed{I = 1.96605}$ ... approx.

iii) Trapezoidal Rule :-

$$I = \frac{h}{2}[y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$$

$\boxed{\therefore I = 2.02142}$ ... approx.

Pro: A solid revolution is formed by rotating about x axis the area betn x axis, the lines $x = 0$, & $x = 1$ & a curve through the points.

x	0.00	0.25	0.50	0.75	1.00
y	1.000	0.9896	0.9589	0.9089	0.8415

Evaluate the vol. of solid formed.

Soln :- If V is the vol. of solid, we have,

$$V = \pi \int_0^1 y^2 \cdot dx$$

To evaluate y.

	y_0	y_1	y_2	y_3	y_4
x	0.00	0.25	0.50	0.75	1.00
y	1.000	0.9896	0.9589	0.9089	0.8415
y^2	1.000	0.9793	0.9195	0.8261	0.7081

Here $h = 0.25$, As $n = 5$, Use simpson's $1/3^{rd}$ Rule,

$$\therefore V = \frac{0.25\pi}{3}[1.000 + 0.7081 + 4(0.9793 + 0.8261) + 2(0.9195 \cancel{+ 0.7081})]$$

~~V = 6~~ $\boxed{V = 2.8192 \text{ Cubic Units}}$ ... approx.

Using Trapezoidal Rule ;

$$V = \frac{0.25\pi}{2}[1 + 0.7081 + 2(0.9793 + 0.9195 + 0.8261)]$$

$\boxed{\therefore V = 2.8109}$ ... approx.

Not applicable :-
Simpsons $3/8^{th}$ Rule,

$$V = 0.25 \cdot \pi \times \frac{3}{8}[1 + 0.7081 + 3(0.9793 + 0.9195 + 0.7081) + 2(0.8261)]$$

~~V = 6~~ $\boxed{V = 3.2931}$ ... approx

Pro:- Evaluate : $I = \int_0^{\pi/4} \cos x \, dx$ by dividing the interval into 3 strips.

Sol^n: at $x_0 = 0$, $x_n = \pi/4$.

$\therefore$ Dividing area in 3 strips, $\left(\frac{x_0 + x_n}{n}\right) = h = \left(\frac{0 + \pi/4}{3}\right) = \frac{\pi}{12}$

i.e.	$x_0 = 0$,	$x_1 = \frac{\pi}{12}$	$x_2 = \frac{\pi}{6}$	$x_3 = \frac{\pi}{4}$.
$y = \cos x$	$y_0 = 1$	$y_1 = 0.9659$	0.8660	$y_3 = 0.7071$
	y_0	y_1	y_2	y_3

$\therefore$ Simpsons $3/8^{th}$ Rule :-

$\therefore I = \frac{3}{8}\left(\frac{\pi}{12}\right)\left[\cancel{y_0 + y_3 + 3(}\; y_0 + y_3 + 3(y_1 + y_2)\right]$

$= \frac{3}{8}\left(\frac{\pi}{12}\right)\left[1 + 0.7071 + 3(0.9659 + 0.8660)\right]$

$\therefore \boxed{I = 0.7071}$ approx

Not applicable

Simpsons $1/3^{rd}$ Rule :-

$I = \frac{h}{3}\left[y_0 + y_3 + 3(y_1) + 2(y_2)\right]$

$= \frac{\pi/12}{3}\left[1 + 0.7071 + 3 \times 0.9659 + 2 \times 0.866\right]$

$\therefore \boxed{I = 0.5530}$ .. approx

Trapezoidal Rule :

$I = \frac{\pi/12}{2}\left[1 + 0.7071 + 2(0.9659 + 0.866)\right]$

$\therefore \boxed{I = 0.7030}$ ... approx.

V. Imp. Note:- For Simpson's $3/8^{th}$ Rule, $n = (3r + 1)$

i.e. $n = 4, 7, 10, 13, 16, 19, 22, \ldots$

For Simpson's $1/8^{th}$ Rule, $n = \text{odd}$.

i.e. $n = 3, 5, 7, 9, 11, 13, 15, \ldots$

For Trapezoidal Rule, $n = \text{Any No.}$

i.e. $n = 2, 3, 4, 5, 6, 7, \ldots$

Pro: Calculate $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x) \cdot dx$ by using ~~u~~ simpson's $3/8^{th}$ Rule. Take $h = 0.1$

$\log_e$ ↳ $\underline{\ln}$

Let, ~~I~~

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
$y = (\sin x - \log_e x + e^x)$	3.0295	2.8494	2.7975	2.8213	2.8976	3.0146	3.1660	3.3483	3.5598	3.8001	4.0698	4.3705	4.7042
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}	y_{12}

$$\therefore I = \frac{3}{8} h \left[y_0 + y_{12} + 3(y_1 + y_2 + y_4 + y_5 + y_7 + y_8 + y_{10} + y_{11}) + 2(y_3 + y_6 + y_9) \right]$$

$$= \frac{3 \times 0.1}{8} \left[3.0295 + 4.7042 + 3(2.8494 + 2.7975 + 2.8976 + 3.0146 + 3.3483 + 3.5598 + 4.0698 + 4.3705) + 2(2.8213 + 3.1660 + 3.8001) \right]$$

$\boxed{I = 4.0512}$ ... approx.

Pro:- Evaluate $\int_0^3 \frac{dx}{1+x}$ with 7 ordinates by using simpson's $3/8^{th}$ rule & hence calculate log2. ~~Estimate~~

It is given that $n = 7$.

$\therefore$ We can use Simpson's $3/8^{th}$ rule or $1/3^{rd}$ Rule. $\therefore h = \frac{(0+3)}{n-1} = 0.5$

x	0	0.5	1.0	1.5	2.0	2.5	3.0
$y = \frac{1}{1+x}$	1.000	0.6667	0.5000	0.4000	0.3333	0.2857	0.2500
	0	1	2	3	4	5	6

$$\therefore I = \frac{3}{8} h \left[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3) \right]$$

$$= \frac{3}{8} \times 0.5 \left[1.000 + 0.2500 + 3(0.6667 + 0.5000 + 0.3333 + 0.2857) + 2(0.4) \right]$$

$\therefore \boxed{I = 1.3888}$ ... approx.

By direct Integration, $I = \int_0^3 \frac{dx}{1+x} = [\log(1+x)]_0^3 = \log 4.$

$\therefore \log 4 = 1.3888$

$\therefore \log 2^2 = 1.3888$

$\therefore 2 \log 2 = 1.3888$

$\therefore \boxed{\log 2 = 0.6944}$

Pro: Evaluate $\int_0^{3\pi/20} (1+2\sin x)\,dx$ using Simpson's $1/3^{rd}$ rule. Take 4 segments compare this value with analytical result & compare percentage relative error.

Sol^n:- We take 4 segment.

i.e. $x_0 = 0$, $x_4 = \frac{3\pi}{20}$

$$\frac{(x_0+x_4)}{4} = h = \frac{\frac{3\pi}{20}}{4} - h \quad \therefore h = \frac{3\pi}{80} \quad ; n = 5$$

x_0	0	$\frac{3\pi}{80}$	$\frac{3\pi}{40}$	$\frac{9\pi}{80}$	$\frac{3\pi}{20}$
$y = 1+2\sin x$	1	1.2351	1.4669	1.6922	1.9080
	y_0	y_1	y_2	y_3	y_4

$$\therefore I = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$$

$$= \frac{\pi}{80}\left[(1+1.9080) + 4(1.2351+1.6922) + 2(1.4669)\right]$$

$\boxed{I = 0.6874}$

Actual value of $I = \int_0^{3\pi/20} (1+2\sin x)\,dx$

$$= \left[x + (-2\cos x)\right]_0^{3\pi/20} = \left[\frac{3\pi}{20} - 2\cos\left(\frac{3\pi}{20}\right) - (0 - 2\cos 0)\right]$$

$\therefore \boxed{I_{act} = 0.6892}$

$$\%\ \text{error} = \frac{0.6892 - 0.6874}{0.6892}$$

$$= 0.2649\ \%$$

Pro: Evaluate the integral, $\int_0^{\pi} (4+2\sin x)\,dx$ using Simpsons $3/8^{th}$ rule where $n=5$, compute percentage relative error.

Sol^n:- let $x_0 = 0$, $x_n = \pi$, i.e. $x_5 = \pi$

$$\therefore \left(\frac{x_0+x_5}{5}\right) = h = \frac{0+\pi}{5} \quad \therefore h = \frac{\pi}{5}$$

$\therefore x$	0	$\pi/5$	$2\pi/5$	$3\pi/5$	$4\pi/5$	π
$y = 4+2\sin x$	4.0000	5.1756	5.9021	5.9021	5.1756	4.0000
	y_0	y_1	y_2	y_3	y_4	y_5

$\therefore I = \frac{3h}{8}\left[y_0 + y_5 + 3(y_1 + y_2 + y_4) + 2y_3\right]$

$= \frac{3}{8}\left(\frac{\pi}{5}\right)\left[4 + 4 + 3(5.1756 + 5.9021 + 5.1756) + 2(5.9021)\right]$

$\therefore \boxed{I = 16.1550}$

$$I_{act} = \int_0^{\pi} (4 + 2\sin x)\,dx$$

$= \left[4x - 2\cos x\right]_0^{\pi} = \left[4\pi - 2\cos\pi + 2\cos 0\right]$

$\therefore \boxed{I_{act} = 16.5664}$

$\therefore$ % Relative error $= \frac{I_{act} - I}{I_{act}} \times 100$

$= \frac{16.5664 - 16.1550}{16.5664}$

$= 2.4833$ %

<u>Pro</u>:- If $y = a + bx + cx^2$ then show that $\int_1^3 y\,dx = 2y_2 + \frac{1}{12}(y_0 - 2y_2 + y_4)$ & hence find approx. value of $\int_{-1/2}^{1/2} e^{-x^2/2}\,dx$

Sol^n:- $I = \int_1^3 y \cdot dx$

If shift origin by two x places i.e. put $x = t - 2$

$\therefore \frac{dx}{dt} = \frac{d}{dt}(t-2) = 1$ $\quad \therefore \cancel{t = x+2}$

$\therefore dx = dt$

$\therefore$ at $t = 1$, $x = 1 - 2 = -1$

at $t = 3$, $x = 3 - 2 = 1$

$\therefore I = \int_{-1}^{+1} y \cdot dt = \int_{-1}^{+1} y \cdot dx$ $\quad \because$ changing variables.

It is given that $\int_1^3 y \cdot dx = \int_{-1}^{1} y\,dx = 2y_2 + \frac{1}{12}(y_0 - 2y_2 + y_4)$

i.e. at $t = 2$, $x = 2 - 2 = 0 \rightsquigarrow y_2 \Rightarrow y_0$

$t = 0$, $x = -2 \rightsquigarrow y_0 \Rightarrow y_{-2}$

$t = 4$, $x = 4 - 2 = 2 \rightsquigarrow y_4 \Rightarrow y_2$

$\therefore \int_1^3 y \cdot dx = \int_{-1}^{1} y \cdot dx = 2y_0 + \frac{1}{12}(y_{-2} - 2y_0 + y_2)$ .... (a)

It is given that, $y = a + bx + cx^2$

$$\therefore \int_{-1}^{1} y \cdot dx = \int_{-1}^{1} (a+bx+cx^2)\cdot dx = \left[ax + b\frac{x^2}{2} + c\frac{x^3}{3}\right]_{-1}^{1}$$

$$\therefore L.H.S. = \left[a(1) + \frac{b(1)^2}{2} + \frac{c(1)^3}{3} - \left(-a + \frac{b}{2} - \frac{c}{3}\right)\right]$$

$$= a + a + \frac{b}{2} - \frac{b}{2} + \frac{c}{3} + \frac{c}{3} = 2\left(a + \frac{c}{3}\right)$$

Now, $y_0 = a + bx + cx^2$

$$\therefore y_0 = a + b(0) + c(0)^2 = a$$

$$y_{-2} = a - 2b + c(4) = a - 2b + 4c$$

$$y_2 = a + 2b + 4c$$

$$\therefore R.H.S = 2y_0 + \frac{1}{12}(y_{-2} - 2y_0 + y_2)$$

$$= 2a + \frac{1}{12}[(a-2b+4c) - 2(a) + (a+2b+4c)]$$

$$= 2a + \frac{1}{12}[\cancel{a} - \cancel{2b} + 4c - \cancel{2a} + \cancel{a} + \cancel{2b} + 4c]$$

$$= 2a + \frac{1}{12}(8c) = 2a + \frac{2}{3}c = 2\left(a + \frac{c}{3}\right) = L.H.S.$$

$$\therefore RHS = L.H.S.$$

$$\boxed{\int_1^3 y\,dx = 2y_2 + \frac{1}{12}(y_0 - 2y_2 + y_4).}$$

$\boxed{x = t - 2}$

Let $y = e^{-x^2/2}$ $\quad\quad \therefore$ at $t = -1/2$, $x =$

$$I = \therefore \int_{-1/2}^{1/2} e^{-x^2/2} \cdot dx = \frac{1}{2}\left\{2y_0 + \frac{1}{12}(y_{-1} - 2y_0 + y_1)\right\} \quad \cdots \text{from (a)}$$

$$\therefore y_0 = e^{-0} = 1$$

$$y_{-1} = e^{1/2} = e^{1/2} = \cancel{7.3891}\ 1.6487$$

$$y_2 = e^{-1/2} = e^{-1/2} = \cancel{0.1353}\ 0.6065$$

$$\therefore I = \frac{1}{2}\left\{2(1) + \frac{1}{12}[\cancel{7.3891}\ 1.6487 - 2(1) + \cancel{0.1353}\ 0.6065]\right\}$$

$$I = \cancel{2.4604/2 = 1.2302}\ 1.0106$$

$$\therefore \int_{-1/2}^{1/2} e^{-x^2/2} dx = \cancel{2.4604}\ \cancel{1.2302}\ 1.0106$$

$\boxed{act = 0.9599}$

Pro: Use trapezoidal rule to calculate, $I = \int_0^{\pi} e^x \cdot \cos x \, dx$

Consider $n = 8$, $\left(\frac{0+\pi}{8}\right) = h \Rightarrow h = \pi/8$

x	0	$\pi/8$	$2\pi/8$	$3\pi/8$	$4\pi/8$	$5\pi/8$	$6\pi/8$	$7\pi/8$	π
$y = e^x \cos x$	1.000	1.3682	1.5509	1.2430	0	−2.7263	−7.4605	−14.4359	−24.1407
	0	1	2	3	4	5	6	7	8

$\therefore I = \frac{h}{2}\left[y_0 + y_8 + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$

$= \frac{1}{2}\left(\frac{\pi}{8}\right)\left[1 - 24.1407 + 2(1.3682 + 1.5509 + 1.243 + 0 - 2.7263 - 7.4605 - 14.4359)\right]$

$\boxed{I = -12.5785}$

Pro: Use Simpson's 3/8 rule to integrate the f^n $f(x) = 0.2 + 20x + 25x^2 + 60x^3$ over the limits $a = 0$, $b = 10$.

$\cancel{h = \frac{x_0 + x_n}{n} = \frac{0+10}{10} = 1}$.

x ~~0 1 2 3 4 5 6 7 8 9~~ 10

y For using Simpsons Rule, $(3r+1) = n$

Assuming $n = 10$.

$\therefore \frac{0+10}{n-1} = h \Rightarrow h = \frac{10}{9} = 1.1111$

x	0	1.1111	2.2222	3.3333	4.4444	5.5555	6.6666	7.7777	8.8888	9.9999 ≃ 10
y	0.2	135.5876	826.5148	1003.0547	1241.3217	~~1541.316~~ 11081.847	~~1403.0374~~ 18915.48	~~2326.486~~ 61541.63	~~2811.6616~~ 62009.024	~~3358.5645~~ 62538.1496
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9

$\boxed{\therefore I = 159335.3594}$