Theory of Structures

Notes by-

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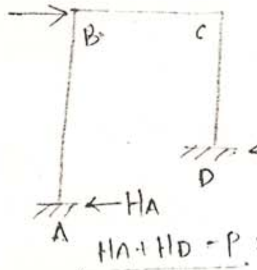
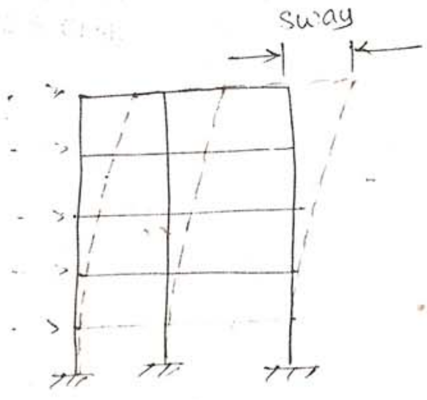


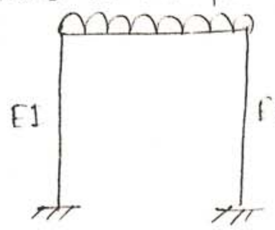
3 Unknown forces.
3 Unknown displacement.

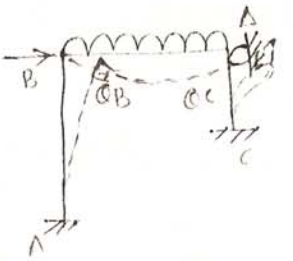
$H_A + H_D = P$:- column shear



Sway is nothing but the lateral (unknown) oint translations.

The frame will not sway only if both geometry & loading are symmetrical. & when support conditions do not permit for sway.





"Sway is feature of Only frame"
Sway is Lateral Translation.

~~15~~ Rotation.
5 Translation
20 ~~19~~ Unknown Displacement

Static Indeterminacy :- Thumb Rule.
[Count the No. of beam X3]
In case of internal hinge, No. of beam X 3 - 1
∴ 30 Unknown forces.

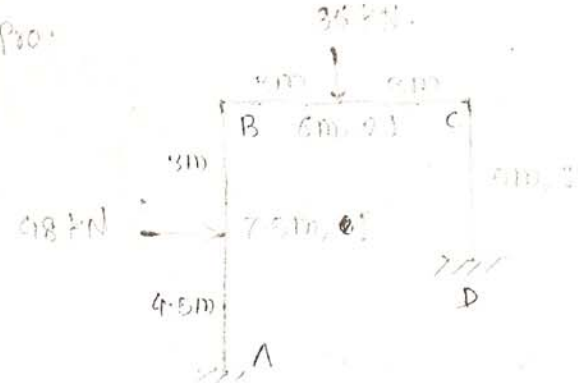
∴ Disp. mtd. is suitable. & SP is preferable.
SP mtd. is ideally suited mtd for frame analysis. Since the unknown disp. in general will be far less than the unknown forces.

It is extremely compatable for machine. So in the Software packages SDM is generally used.

When sway is accounted for the '...3R' term in the SD eq^n, will come in to play while considering the columns, not for the beam as beams are in translat^n

The equilibrium eq^{ns}, as in the case of beams, and the consideration of 'column shear' will be sufficient to solve for the SD eq^n.

Pro.



Unknown,

θ_B, θ_C

$\Delta \rightarrow$ assumed Rightwards.

① $M_{AB} = -\frac{48 \times 4.5 \times 3^2}{7.5^2} + \frac{2EI}{l}\left[4\theta_A + 2\theta_B - 3\right.$

① $M_{AB} = -34.56 + \frac{2(EI)}{7.5}\left[2\theta_B - \frac{3\Delta}{7.5}\right]$

$\therefore M_{AB} = -34.56 + 0.533\theta_B - 0.1067\Delta$

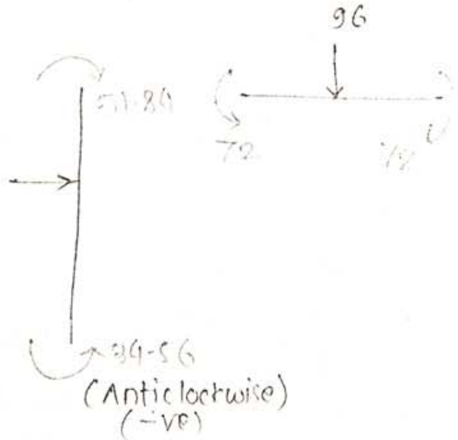
② $M_{BA} = +51.84 + \frac{2EI}{7.5}\left(2\theta_B + 0 - \frac{3\Delta}{7.5}\right)$

③ $M_{BC} = -72 + \frac{4EI}{6}(2\theta_B + \theta_C)$

④ $M_{CB} = +72 + \frac{4EI}{6}(\theta_B + 2\theta_C)$

⑤ $M_{CD} = 0 + \frac{2EI}{5}\left(2\theta_C + 0 - \frac{3\Delta}{5}\right)$

⑥ $M_{DC} = 0 + \frac{2EI}{5}\left(\theta_C + 0 - \frac{3\Delta}{5}\right)$



We have, $M_{BA} + M_{BC} = 0$

$M_{CB} + M_{CD} = 0.$

$\therefore -20.16 + \frac{2EI}{7.5}\left(2\theta_B - \frac{3\Delta}{7.5}\right) + \frac{4EI}{6}(2\theta_B + \theta_C) = 0$

$\therefore -20.16 + 1.867\theta_B EI + 0.67\theta_C EI - 0.1067\Delta EI = 0$

$\therefore 1.867EI\theta_B + 0.67EI\theta_C - 0.1067EI\Delta = 20.16$ — (A)

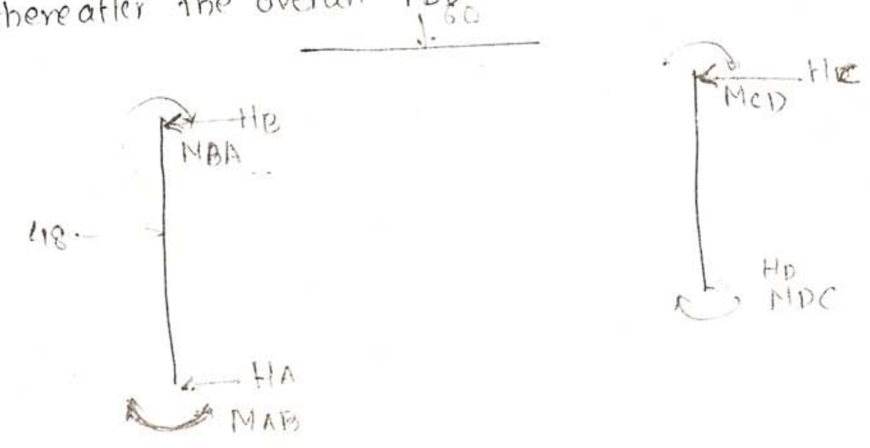
$72 + 0.67EI\theta_B + 0.8$

$\therefore 1.8\dot{6}EI\theta_B + 0.6\dot{6}EI\theta_C - 0.10\dot{6}EI\Delta = 20.16$ — (A)

$0.\dot{6}EI\theta_B + 2.1\dot{3}EI\theta_C - 0.24EI\Delta = -72$ — (B)

To generate 3rd eqⁿ, we consider the FBD of the columns.

& thereafter the overall FBD.



$\therefore M_{AB} + M_{BA} - 48\times 3 + 7.5\,H_A = 0 \qquad M_{CD} + M_{DC} + 5\,H_D = 0$

$\& \; \boxed{H_A + H_D = 48} \rightarrow$ eqⁿ of column shear.

$$\therefore H_A = \frac{144 - (M_{AB} + M_{BA})}{7.5} \qquad \& \; H_D = \frac{-(M_{CD} + M_{DC})}{5}$$

$$\& \quad \frac{144 - (M_{AB} + M_{BA})}{7.5} + \left[\frac{-(M_{CD} + M_{DC})}{5}\right] = 48.$$

$$\therefore 5400 - \cancel{37.5}\,M_{AB} - \cancel{37.5}\,M_{BA} - 37.5\,M_{CD} - 7.5\,M_{DC} = 1800.$$

$$\boxed{-0.106\,EI\theta_B - 0.24\,EI\theta_B + 0.124\,EI\Delta = 31.1} \quad \text{(C)}$$

Solving (A), (B), (C),

$EI\theta_B = 30.45$

$EI\theta_C = -15.5$

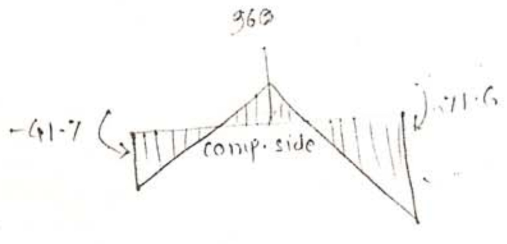
$EI\Delta = 247.$

columns at matrix & Row of matrix should be same: symmetrical.

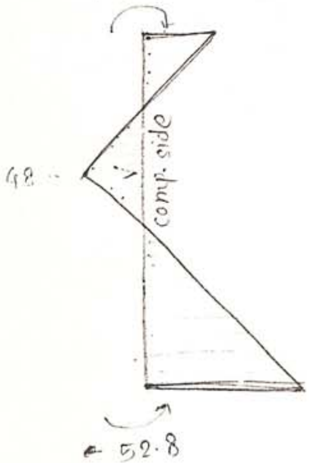
$$\boxed{a_1x_1 + by_1 + c_1z_1 = d_1}$$
$$a_2x_2 + b_2y_2 + c_2z_2 = d_2$$
$$a_3x + b_3y + c_3z = d_3.$$

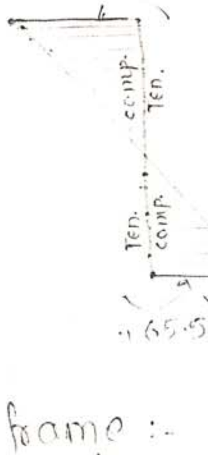
Put back:-

$M_{AB} = -52.8$ kNm
$M_{BA} = +41.7$ kNm
$B_{BC} = -41.7$ kNm
$M_{CB} = +71.6$ kNm
$M_{CD} = -71.6$ kNm
$M_{DC} = -65.5$ kNm



See direction of BMD arrow.





arrows shows comp. side

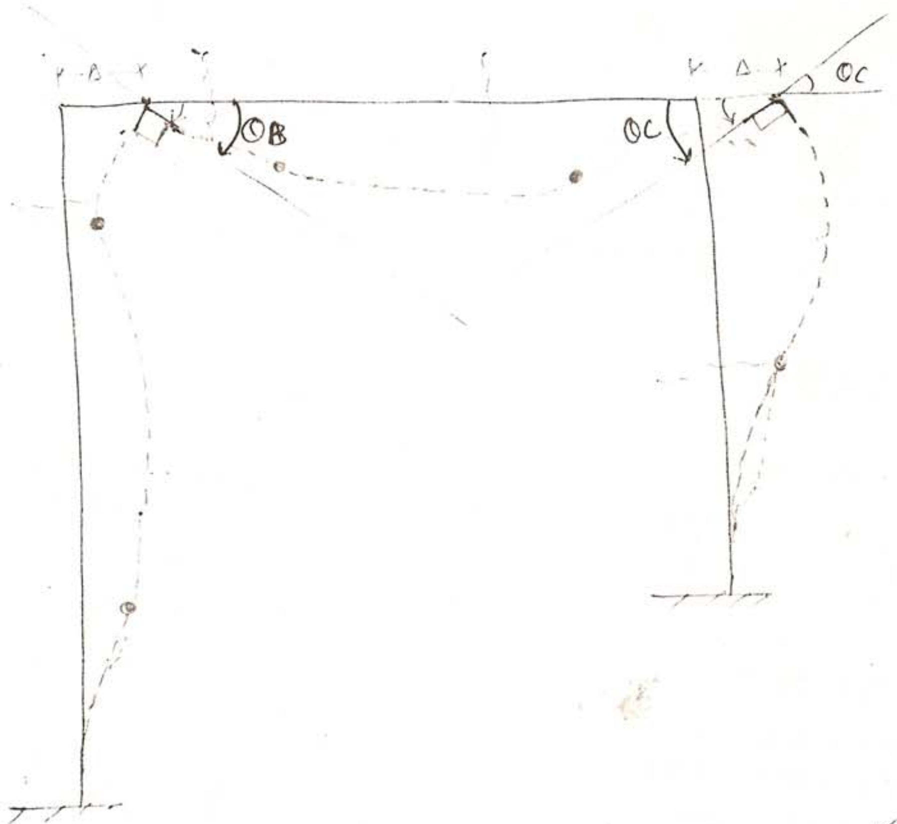
BM for frame:-
These are no +ve or -ve sign assigned for BMD*. Draw BMD on compression side.

Elastic curve:

Rotates clockwise ~ $EI\theta_B = +30.45$

Rotates anticlockwise ~ $EI\theta_C = -15.5$

$EI\Delta = 26?$



$$M_{AB} = \frac{4EI\theta_A}{3} + \frac{2EI\theta_B}{L}$$

$$\frac{2}{3}\left(\frac{12.5}{8}\right) + \frac{4}{3}\left(\frac{118.73}{8}\right)$$